Calculus A Final Review Worksheet #1 Name: Key

1. $\lim_{x\to -3}(-2+x)^3 = (-2-3)^3$

$= \boxed{-125}$

2. $\lim_{x\to -2}\dfrac{x^2+x-2}{2+x} = \lim_{x\to -2}\dfrac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}}$

$= -2-1$

$= \boxed{-3}$

3. Using the definition of derivative find the derivative of $y = x^2 - 2x$.

$\lim_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x} \rightarrow \lim_{\Delta x\to 0}\dfrac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2-2x)}{\Delta x}$

$= \lim_{\Delta x\to 0}\dfrac{1}{\Delta x}\left[x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - x^2 + 2x\right]$

$= \lim_{\Delta x\to 0}\dfrac{1}{\Delta x}\left[\Delta x(2x+\Delta x - 2)\right]$

$= \boxed{2x-2}$

4. Find $\dfrac{dy}{dx}$ for $y = \sqrt[3]{x^2}(x+1) = x^{2/3}(x+1)$ product rule

$\dfrac{dy}{dx} = x^{2/3}(1) + \frac{2}{3}x^{-1/3}(x+1)$

$= x^{2/3}\cdot\dfrac{3x^{1/3}}{3x^{1/3}} + \dfrac{2x+2}{3x^{1/3}}$

$= \boxed{\dfrac{5x+2}{3\sqrt[3]{x}}}$

or → distribute first: $y = x^{5/3} + x^{2/3}$

$\dfrac{dy}{dx} = \frac{5}{3}x^{2/3} + \frac{2}{3}x^{-1/3}$ then factor

$= \dfrac{x^{-1/3}}{3}(5x+2)$

$= \boxed{\dfrac{5x+2}{3\sqrt[3]{x}}}$

5. Find $\dfrac{d^2y}{dx^2}$ for $y = \dfrac{x+4}{x-1}$

$\dfrac{dy}{dx} = \dfrac{(x-1)(1)-(x+4)(1)}{(x-1)^2}$

$= -\dfrac{5}{(x-1)^2}$

$= -5(x-1)^{-2}$

$\dfrac{d^2y}{dx^2} = 10(x-1)^{-3}(1)$

$= \boxed{\dfrac{10}{(x-1)^3}}$

6. Find y' for $2y^2 - 2xy = x^3$

$4y\frac{dy}{dx} - 2x\frac{dy}{dx} - 2y = 3x^2$

$\frac{dy}{dx}(4y-2x) = 3x^2 + 2y$

$\boxed{\dfrac{dy}{dx} = \dfrac{3x^2+2y}{4y-2x}}$

7. Find the equation for the tangent line of $y = 2x^2 + x - 2$ at the point where $x = 2$ $\rightarrow$ $y = 2(4) + 2 - 2$

$y = 8$

$y' = 4x + 1$

slope @ $x = 2$

$m = 4(2) + 1$

$= 9$

$\boxed{y - 8 = 9(x - 2)}$

8. Suppose the position equation for a moving object is given by $s(t) = -16t^2 - 4t + 1$. Find the velocity when $t = 1$

$v(t) = s'(t) = -32t - 4$

$v(1) = -32 - 4$

$\boxed{v(1) = -36}$

9. Find the <u>absolute</u> maximum and absolute minimum of f on the interval $(0,2)$, $f(x) = \dfrac{x^3 - x^2 - 2x}{x}$

first simplify:

$f(x) = x^2 - x - 2$

$f'(x) = 2x - 1$

$0 = 2x - 1$

critical # @ $x = \frac{1}{2}$

f': 0 $\quad - \quad \frac{1}{2} \quad + \quad$ 1

$\searrow \quad \nearrow$

$\boxed{\text{Absolute Minimum @ } \left(\tfrac{1}{2}, -\tfrac{9}{4}\right)}$

open interval, so no end points to check.

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8} - \frac{1}{4} - 1\right)$

$= -\frac{9}{4}$

10. The radius of a circle is measured at 4 inches. If the measurement is correct to within .02 inch, use differentials to estimate the propagated error in the area of the circle.

$dr = (\pm 0.02)$

$A = \pi r^2$

$\dfrac{dA}{dr} = 2\pi r$

$dA = 2\pi r\, dr$

$dA = 2\pi(4)(\pm 0.02)$

$dA = \pm 0.16\pi$

$\boxed{\approx \pm 0.503}$

11. Find $f'(x)$ for $f(x) = \dfrac{1}{2x - e^{2x}}$ $\rightarrow$ $f(x) = (2x - e^{2x})^{-1}$

$f'(x) = -\dfrac{2 - 2e^{2x}}{(2x - e^{2x})^2}$

$\boxed{f'(x) = \dfrac{2e^{2x} - 2}{(2x - e^{2x})^2}}$

12. Differentiate $f(x) = x^{3x^2}$

$\ln y = 3x^2 \ln x$

$\dfrac{1}{y}\dfrac{dy}{dx} = 3x^2 \dfrac{1}{x} + 6x \ln x$

$\dfrac{1}{y}\dfrac{dy}{dx} = 3x(1 + 2\ln x)$

$\dfrac{dy}{dx} = 3x(1 + 2\ln x)\, x^{3x^2}$

$\boxed{\dfrac{dy}{dx} = 3x^{3x^2+1}(1 + 2\ln x)}$

13. Take the derivative: $y = \ln \dfrac{x^3 \sqrt[5]{x^2 - x}}{(x^2 - 1)^2}$

Expand:

$y = 3\ln x + \frac{1}{5}\ln(x^2 - x) - 2\ln(x^2 - 1)$

LCD: $5x(x^2 - 1)$

$y' = \dfrac{3}{x} + \dfrac{2x - 1}{5x(x - 1)} - \dfrac{4x}{x^2 - 1} \cdot \dfrac{5x}{5x}$

$y' = \dfrac{15(x^2 - 1) + (2x - 1)(x + 1) - 20x^2}{5x(x^2 - 1)}$

$\boxed{y' = \dfrac{-3x^2 + x - 16}{5x(x^2 - 1)}}$